SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 7 Polynomials and other functions

Exercise 7A

Question 1

a $y = 0^3 + 0^2 + 0 + 1 = 1$ (0, 1)

b $y = 3(0)^3 - 5(0)^2 - 2(0) - 5 = -5$ (0, −5)

c $y = 0^3 + 8 = 8$ (0, 8)

d $y = 2(0)^3 + 3(0)^2 + 6 = 6$ (0, 6)

e $y = 2 + 3(0) + 7(0)^2 - 0^3 = 2$ (0, 2)

f $y = 5(0) + 3 + 2(0)^3 = 3$ (0, 3)

Question 2

a $y = (x-2)(x-3)(x-4)$

$x - 2 = 0$ or $x - 3 = 0$ or $x - 4 = 0$

$x = 2$ $\quad$ $x = 3$ $\quad$ $x = 4$

x-intercepts $(2, 0), (3, 0), (4, 0)$

b $y = (x+7)(x+1)(x-5)$

$x + 7 = 0$ or $x + 1 = 0$ or $x - 5 = 0$

$x = -7$ $\quad$ $x = -1$ $\quad$ $x = 5$

$\therefore$ x-intercepts $(-7, 0), (-1, 0), (5, 0)$

c $y=(2x-5)(x+1)(5x-3)$

$2x-5=0 \quad \text{or} \quad x+1=0 \quad \text{or} \quad 5x-3=0$

$x=\frac{5}{2} \qquad x=-1 \qquad x=\frac{3}{5}$

$\therefore x\text{-intercepts } (\frac{5}{2},0),(-1,0),(\frac{3}{5},0)$

d $0=(1-x)(1+x)(x-7)$

$1-x=0 \quad \text{or} \quad 1+x=0 \quad \text{or} \quad x-7=0$

$x=1 \qquad x=-1 \qquad x=7$

$\therefore x\text{-intercepts} (1,0),(-1,0),(7,0)$

e $0=x(4x-1)(2x-7)$

$x=0 \quad \text{or} \quad 4x-1=0 \quad \text{or} \quad 2x-7=0$

$x=\frac{1}{4} \qquad x=3.5$

$\therefore x\text{-intercepts } (0,0),(\frac{1}{4},0),(3.5,0)$

f $0=(x+1)^2(x-5)$

$x+1=0 \quad \text{or} \quad x-5=0$

$x=-1 \qquad x=5$

$\therefore x\text{-intercepts } (-1,0),(5,0)$

g $0=x^3-9x$

$=x(x^2-9)$

$\therefore x=0 \quad \text{or} \quad x^2-9=0$

$x=\pm 3$

$\therefore x\text{-intercepts } (-3,0),(0,0),(3,0)$

h $y=x^3+2x^2-15x$

$=x(x^2+2x-15)$

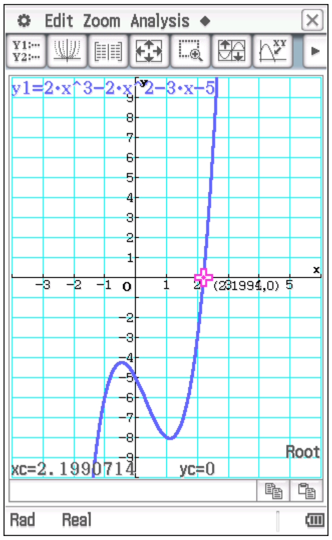
$=x(x+5)(x-3)$

$\therefore x=0 \quad \text{or} \quad x+5=0 \quad \text{or} \quad x-3=0$

$x=-5 \qquad x=3$

$\therefore x\text{ - intercepts } (-5,0),(0,0),(3,0)$

Question 3



Question 4

a $$2\times(-3)\times(-k)=-36$$
$$6k=-36$$
$$k=-6$$

b $$0=(x+2)(x-3)(x+6)$$
$$x+2=0 \quad \text{or} \quad x-3=0 \quad \text{or} \quad x+6=0$$
$$x=-2 \qquad x=3 \qquad x=-6$$
$\therefore$ x-intercepts $(-6,0),(-2,0),(3,0)$

Question 5

$f(x) = x^3 - 6x^2 - x + 6$

a $f(-1) = (-1)^3 - 6(-1)^2 - (-1) + 6$
$= 0$

b $f(1) = 1^3 - 6(1)^2 - 1 + 6$
$= 0$

c $f(2) = 2^3 - 6(2)^2 - 2 + 6$
$= -12$

d $f(6) = 6^3 - 6(6)^2 - 6 + 6$
$= 0$

e $f(x) = x^3 - 6x^2 - x + 6$
$= (x+1)(x-1)(x-6)$

Question 6

a $f(1) = 1^3 - 10(1)^2 + 31(1) - 30$
$= -8$

b $f(2) = 2^3 - 10(2)^2 + 31(2) - 30$
$= 0$

c $f(3) = 3^3 - 10(3)^2 + 31(3) - 30$
$= 0$

$$\begin{aligned}
f(x) &= x^3 - 10x^2 + 31x - 30 \\
&= (x-2)(x-3)(x+k) \\
(-2)(-3)(k) &= -30 \\
k &= -5 \\
f(x) &= x^3 - 10x^2 + 31x - 30 \\
&= (x-2)(x-3)(x-5)
\end{aligned}$$

Question 7

a $3x^3 - 14x^2 - 7x + 10 = (3x - 2)(ax^2 + bx + c)$

$$3x(ax^2) = 3x^2 \therefore a = 1$$

$$(-2) \times c = 10 \therefore c = -5$$

b $(3x - 2)(x^2 + bx - 5) = 3x^3 + 3bx^2 - 15x - 2x^2 - 2bx + 10$

$$3bx^2 - 2x^2 = -14x^2$$

$$3b - 2 = -14$$

$$3b = -12$$

$$b = -4$$

c $y = 3x^3 - 14x^2 - 7x + 10$

$$= (3x - 2)(x^2 - 4x - 5)$$

$$= (3x - 2)(x - 5)(x + 1)$$

x-int, $y = 0$

$0 = (3x - 2)(x - 5)(x + 1)$

$3x - 2 = 0$ or $x - 5 = 0$ or $x + 1 = 0$

$x = \frac{2}{3}$ $\quad x = 5$ $\quad x = -1$

$\therefore$ x-intercepts $(-1, 0), (\frac{2}{3}, 0), (5, 0)$

Question 8

a $x\text{-int}: \ (-2,0)(2,0)(5,0)$

$y\text{-int}: \ 2x(-2)\times(-5)=20 \quad (0,20)$

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$

b $x\text{-int}: \ (-4,0)(-1,0)(5,0)$

$y\text{-int}: \ 4\times1\times(-5)=-20 \quad (0,-20)$

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$

c $x\text{-int}: \ (-4,0)(-1,0)(5,0)$

$y\text{-int}: \ 2\times4\times1\times(-5)=-40 \quad (0,40)$

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$

d $x\text{-int}: \ (0,0)(3,0)(7,0)$

$y\text{-int}: \ 0\times3\times(-7)=0 \quad (0,0)$

$x\to\infty,\ y=(\text{large +ve})\times(\text{large -ve})\times(\text{large +ve})$

$=$ very large negative

e $x\text{-int}: \ (1,0)(3,0)$

$y\text{-int}: \ (-1)(-3)(-3)=-9 \quad (0,-9)$

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$

f $x\text{-int}: \ (2,0)$

$y\text{-int}: \ (0,8)$

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$

Exercise 7B

Question 1

B: translated right by 3 units $\therefore y = \sqrt{(x-3)}$

C: translated 4 units up $\therefore y = \sqrt{x} + 4$

D: translated 3 units left, 5 units down $\therefore y = \sqrt{(x+3)} - 5$

Question 2

a $y = \dfrac{1}{x} + 1$

b $y = \dfrac{1}{x} + 2$

c $y = \dfrac{1}{x} - 1$

Question 3

a Translated 1 unit left : $\dfrac{1}{x+1}$

b Translated 3 units right : $\dfrac{1}{x-3}$

c Translated 1 unit right : $\dfrac{1}{x-1}$

Question 4

The graph of $y = x^3 + 1$ is that of $y = x^3$ translated up 1 unit.

Question 5

The graph of $y = \dfrac{1}{x-1}$ is that of $y = \dfrac{1}{x}$ translated 1 unit to the right.

Question 6

The graph of $y = 2\sqrt{x}$ is that of $y = \sqrt{x}$ dilated parallel to the y-axis, scale factor 2.

Question 7

The graph of $y = (x-3)^2$ is that of $y = (x+4)^2$ translated 7 units right.

Question 8

The graph of $y = \sqrt{x-2}+1$ is that of $y = \sqrt{x}$ translated 2 units right and 1 unit up.

Question 9

The graph of $y = \dfrac{3}{x-1}$ is that of $y = \dfrac{1}{x}$ translated 1 unit to the right and dilated parallel to the y-axis, scale factor 3.

Question 10

a B, F

b D

c C, E, G, H

d H

e C $\rightarrow$ E, G $\rightarrow$ H

f A $\rightarrow$ C, E $\rightarrow$ G, H $\rightarrow$ I

Question 11

A(0, 10), B(−0.51, 0), C(3.08, 0), D(6.42, 0), E(1, 17), F(5, −15), G(3, 1)

Question 12

a $P = \dfrac{400}{V} \rightarrow V = \dfrac{400}{P}$

$V = \dfrac{400}{40}$

$= 10$

When $P = 40$, $V = 10$.

b $V = \dfrac{400}{P}$

$V = \dfrac{400}{20}$

$= 20$

When $P = 20$, $V = 20$.

c Volume cannot be negative. With a non-zero mass there must be some volume.

Thus $V > 0$ would be a suitable domain for V.

Question 13

Graphs in the top row

The left graph is a cubic which has been translated vertically $\Rightarrow y = x^3 + 8$.

A is the y-intercept, $x = 0$

$y = 0^3 + 8 = 8$

A (0,8)

B is the x-intercept, $y = 0$

$0 = x^3 + 8$

$x^3 = -8$

$x = -2$

B $(-2, 0)$

Centre graph is a quadratic translated 2 units right and 3 unit up $\Rightarrow y=(x-2)^2+3$.

Matching equation is $y=(x-d)^2+e$, giving $d=2, e=3$.
C is y-intercept, $x=0$
$y=(0-2)^2+3=7$
C $(0,7)$

Right graph is $y=\sqrt{x}$ translated left 4 units $\Rightarrow y=\sqrt{x+4}$.

Matching equation is $y=\sqrt{x+a} \Rightarrow a=4$.

D is the y-intercept, $x=0$
$y=\sqrt{0+4}=2$
D $(0,2)$

Graphs in the middle row

Left graph shows a reciprocal graph translated right 1 unit and up $\Rightarrow y=\dfrac{1}{x-1}+g$.

Using (0, 2), $2=\dfrac{1}{0-1}+g \Rightarrow g=3$

$$y=\frac{1}{x-1}+3$$

E is the x-intercept, $y=0$

$$0=\frac{1}{x-1}+3$$
$$\frac{1}{x-1}=-3$$
$$x-1=-\frac{1}{3}$$
$$x=\frac{2}{3}$$

E $(\frac{2}{3},0)$

Centre graph is a cubic which has been translated up and to the left $\Rightarrow y=(x+?)^3+?$.

Matching equation is $y=(x+1)^3+8$

F is the x-intercept, $y=0$

$$\begin{aligned} 0 &= (x+1)^3+8 \\ (x+1)^3 &= -8 \\ x+1 &= -2 \\ x &= -3 \end{aligned}$$

F $(-3,0)$

G is the y-intercept, $x=0$

$$\begin{aligned} y &= (0+1)^3+8 \\ &= 9 \end{aligned}$$

G $(0,9)$

Right graph is a quadratic which has been translated vertically in a positive direction $\Rightarrow y=x^2+4$.

Matching equation is $y=x^2+4$

H is the y-intercept, H$(0,4)$

Graphs in the bottom row

Left graph is a quadratic which has been translate 4 units to the right $\Rightarrow y=(x-4)^2$.

The y-intercept of $y=(x-4)^2$ should be $(0, 16)$ but the graph shows $(0,8)$.

Our equation is then $y=\frac{1}{2}(x-4)^2$.

Matching equation $y=b(x-c)^2$ which means $b=\frac{1}{2}, c=4$

Centre graph is a linear graph $\Rightarrow y = hx + i$.

Using the intercepts given on the graph:

$(0,3)$
$3 = h(0) + i$
$i = 3$
$(6,0)$
$0 = 6h + 3$
$6h = -3$
$h = -\frac{1}{2}$

Right graph is a reciprocal relationship which has been translated down $\Rightarrow y = \frac{k}{x} - c$.

The only remaining equation is $y = \frac{8}{x} - 2$.

I is the x-intercept, $y = 0$
$0 = \frac{8}{x} - 2$
$\frac{8}{x} = 2$
$x = 4$
I $(4,0)$

Exercise 7C

Question 1

a Reflect $f(x)$ in x-axis.

b Dilate by a factor of $\frac{1}{4}$ horizontally (compressed).

c Dilate by a factor of 4 parellel to y-axis.

Question 2

a $y=-x^2-3x=-(x^2+3x)$

Reflect $y=x^2+3x$ in x-axis.

b Translate $y=x^2+3x$ 5 units down vertically.

c $y=x^2+3x$

Replace x with $\frac{1}{2}x$

$$y=(\frac{1}{2}x)^2+3(\frac{1}{2}x)$$
$$=\frac{x^2}{4}+\frac{3x}{2}$$

$\therefore$ Dilate parallel to x-axis, factor of 2.

Question 3

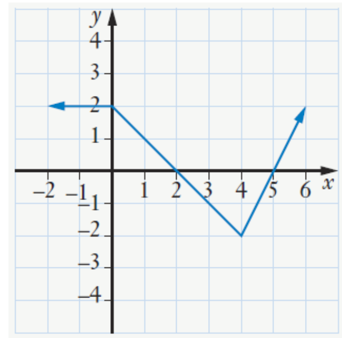
a Translate $y=x^2$ right 3 units.

b Dilation parallel to y-axis, scale factor of 3.

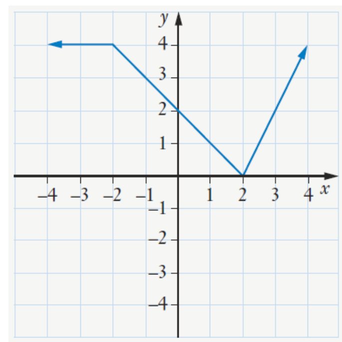
c $y=9x^2$ $\therefore$ Dilation parallel to y-axis, scale factor of 9.

Question 4

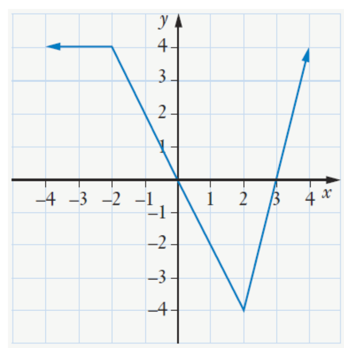
a Translate $f(x)$ 2 units right.



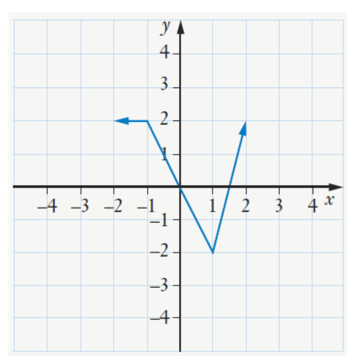
b Translate $f(x)$ 2 units up.



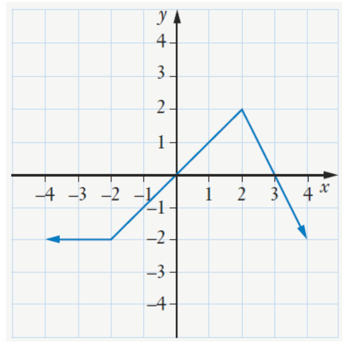
c Dilation parallel to y-axis, scale factor of 2.



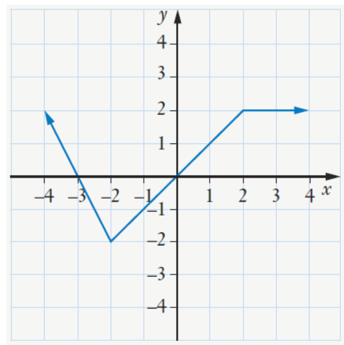
d Dilation parallel to x-axis, scale factor of $\frac{1}{2}$.



e Reflect in x-axis.



f Reflect in y-axis.



Question 5

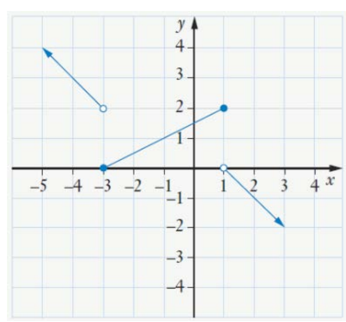
a $f(0)=1$

b $f(1)=1.5$

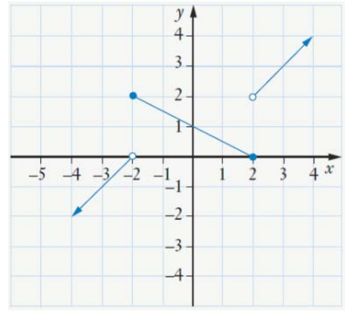
c $f(2)=2$

d $f(-3)=3$

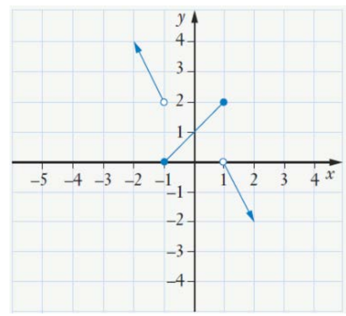
e $f(x+1)$ is $f(x)$ translated 1 unit left.



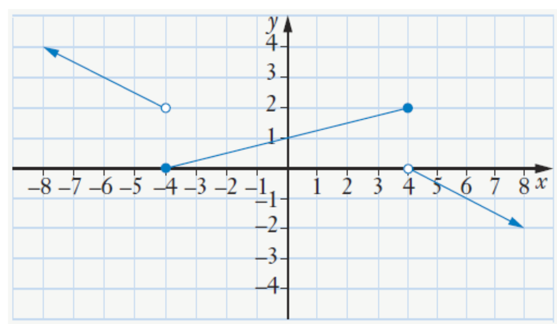
f $y = f(-x)$ is $y = f(x)$ reflected in the y-axis.



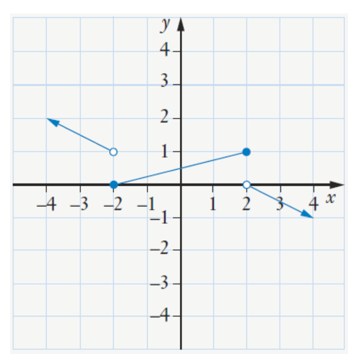
g $y = f(2x)$ is $y = f(x)$ dilated parallel to x-axis, scale factor of $\dfrac{1}{2}$.



h $y = f(0.5x)$ is $y = f(x)$ dilated parallel to x-axis, scale factor of 2.



i $y = 0.5f(x)$ is $y = f(x)$ dilated parallel to the y-axis, scale factor of $\dfrac{1}{2}$.



j $f(1) = 1.5$ from part **b**.

By inspecting the graph of $f(x + 1)$, you can see f(0 + 1), (the value of y when $x = 1$) is also 1.5.

k $f(2) = 2$ from part **c**.

By inspecting the graph of $f(2x)$, you can see $f(2 \times 1)$, (the value of y when $x = 1$) is also 2.

Question 6

A No horizontal shift or scaling.

Vertically $\frac{1}{2}$ the size.

$\therefore y = \frac{1}{2} f(x) \rightarrow \text{III}$

B No vertical dilation or horizontal dilation.

Translated 2 units right.

$\therefore y = f(x-2) \rightarrow \text{X}$

C No vertical or horizontal dilation.

No horizontal translation.

Vertical translation 2 units down.

$\therefore y = f(x) - 2 \rightarrow \text{IX}$

D No vertical dilation.

Horizontal dilation, scale factor of $\frac{1}{2}$.

$\therefore y = f(2x) \rightarrow \text{VI}$

E No vertical or horizontal dilation.

Reflection in x-axis.

$\therefore y = -f(x) \rightarrow \text{I}$

F Reflected in y-axis.

No dilation or translation.

$\therefore y = f(-x) \rightarrow \text{II}$

Question 7

a $f(x-3)$ is $f(x)$ translated 3 units right.

$\therefore$ x-intercepts at (1,0), (7,0), (10,0)

b $f(2x)$ is $f(x)$ dilated parallel to the x-axis, scale factor of $\frac{1}{2}$.

$\therefore$ x-intercepts at $(-1,0)$, (2,0), $(3\frac{1}{2},0)$

c $y=-f(x)$ is $y=f(x)$ reflected in x-axis.

$\therefore$ x-intercepts at $(-2,0)$, (4,0), (7,0)

d $y=f(-x)$ is $y=f(x)$ reflected in y-axis.

$\therefore$ x-intercepts at (2, 0), $(-4,0)$, $(-7,0)$

e $y=f(x)+3$ is $y=f(x)$ translated 3 units up.

$\therefore$ max t.p. at (2,8)

f

$y=-f(x)$ is $y=f(x)$ reflected in x-axis.

$\therefore$ max t.p. at (5,1)

Exercise 7D

Question 1

A, C, D

Question 2

Circle Equation $x^2 + y^2 = 10^2$

Point A : $(-6)^2 + a^2 = 10^2$

$$a^2 = 64$$

$$a = 8$$

Point B : $(3)^2 + b^2 = 10^2$

$$b^2 = 91$$

$$b = \sqrt{91}$$

Point C : $(0)^2 + c^2 = 10^2$

$$c = -10$$

Point D : $(d)^2 + 5^2 = 10^2$

$$\begin{aligned} d^2 &= 75 \\ d &= -\sqrt{75} \\ &= -5\sqrt{3} \end{aligned}$$

Question 3

a $(x-2)^2 + (y+3)^2 = 25$

b $(x-3)^2 + (y-2)^2 = 49$

c $(x+10)^2 + (y-2)^2 = 45$

d $(x+1)^2 + (y+1)^2 = 36$

Question 4

a

$$
\begin{aligned}
(x-3)^2+(y-5)^2 &= 25 \\
x^2-6x+9+y^2-10y+25 &= 25 \\
x^2+y^2-6x-10y &= -9
\end{aligned}
$$

b

$$
\begin{aligned}
(x+2)^2+(y-1)^2 &= 7 \\
x^2+4x+4+y^2-2y+1 &= 7 \\
x^2+y^2+4x-2y+5 &= 7 \\
x^2+y^2+4x-2y &= 2
\end{aligned}
$$

c

$$
\begin{aligned}
(x+3)^2+(y+1)^2 &= 4 \\
x^2+6x+9+y^2+2y+1 &= 4 \\
x^2+y^2+6x+2y+10 &= 4 \\
x^2+y^2+6x+2y &= -6
\end{aligned}
$$

d

$$
\begin{aligned}
(x-3)^2+(y-8)^2 &= (2\sqrt{7})^2 \\
x^2-6x+9+y^2-16y+64 &= 28 \\
x^2+y^2-6x-16y+73 &= 28 \\
x^2+y^2-6x-16y &= -45
\end{aligned}
$$

Question 5

a

$$x^2+y^2=25$$

$$(x-0)^2+(y-0)^2=25$$

centre (0,0)

radius $\sqrt{25}=5$

b

$25x^2+25y^2=9$

$$x^2+y^2=\frac{9}{25}$$

centre (0,0)

radius $\sqrt{\frac{9}{25}}=\frac{3}{5}$

c $(x-3)^2+(y+4)^2=25$

centre $(3,-4)$

radius $\sqrt{25}=5$

d $(x+7)^2+(y-1)^2=100$

centre $(-7,1)$

radius $\sqrt{100}=10$

e $x^2+y^2-6x+4y+4=0$

$(x-3)^2-9+(y+2)^2-4+4=0$

$(x-3)^2+(y+2)^2=9$

centre $(3,-2)$

radius $\sqrt{9}=3$

f $x^2+y^2+2x-6y=15$

$(x+1)^2-1+(y-3)^2-9=15$

$(x+1)^2+(y-3)^2-10=15$

$(x+1)^2+(y-3)^2=25$

centre $(-1,\ 3)$

radius $\sqrt{25}=5$

g $x^2+y^2+2x=14y+50$

$^2+y^2+2x-14y-50=0$

$(x+1)^2-1+(y-7)^2-49-50=0$

$(x+1)^2+(y-7)^2-100=0$

$(x+1)^2+(y-7)^2=100$

centre $(-1,\ 7)$

radius $\sqrt{100}=10$

h $x^2+10x+y^2=151+14y$

$x^2+10x+y^2-14y-151=0$

$(x+5)^2-25+(y-7)^2-49-151=0$

$(x+5)^2+(y-7)^2-225=0$

$(x+5)^2+(y-7)^2=225$

centre $(-5,7)$

radius $\sqrt{225}=15$

i

$$x^2 + y^2 = 20x + 10y + 19$$
$$x^2 - 20x + y^2 - 10y - 19 = 0$$
$$(x-10)^2 - 100 + (y-5)^2 - 25 - 19 = 0$$
$$(x-10)^2 + (y-5)^2 - 144 = 0$$
$$(x-10)^2 + (y-5)^2 = 144$$

centre (10,5)

radius $\sqrt{144} = 12$

j

$$2x^2 - 2x + 2y^2 - 10y = -5$$
$$x^2 - x + y^2 - 5y = -2\frac{1}{2}$$

$$(x - \frac{1}{2})^2 - \frac{1}{4} + (y - 2\frac{1}{2})^2 - 6\frac{1}{4} = -2\frac{1}{2}$$
$$(x - \frac{1}{2})^2 + (y - 2\frac{1}{2})^2 - 6\frac{1}{2} = -2\frac{1}{2}$$
$$(x - \frac{1}{2})^2 + (y - 2\frac{1}{2})^2 = 4$$

centre $(\frac{1}{2},\ 2\frac{1}{2})$

radius $\sqrt{4} = 2$

Question 6

$(x-3)^2 + (y-7)^2 = 36$ centre : (3, 7)

$(x-2)^2 + (y-9)^2 = 49$ centre : (2, 9)

$$\therefore d^2 = (3-2)^2 + (7-9)^2$$
$$= 1 + 4$$
$$d = \sqrt{5}$$

Question 7

$(x-3)^2+(y+4)^2=25 \quad \therefore A\ (3, -4)$

$(x-2)^2+(y-7)^2=9 \quad \therefore B\ (2, 7)$

$$M_{AB}=\frac{7-(-4)}{2-3}$$
$$=-\frac{11}{1}$$

$\therefore y=mx+c$

$m=-11,\ (x,y)=(2,7)$

$\therefore 7=-11(2)+c$

$7=-22+c$

$c=29$

$\therefore y=-11x+29$

Question 8

$(x+1)^2+(y-7)^2=36$

Centre $(-1,\ 7)$ is moved 4 right and 3 down.

New centre $(-1+4,\ 7-3)=(3,\ 4)$

$\therefore (x-3)^2+(y-4)^2=36$

Question 9

$$x^2+y^2-6x+10y+25=0$$
$$(x-3)^2-9+(y+5)^2-25+25=0$$
$$(x-3)^2+(y+5)^2=9$$

Centre $(3,-5)$ is moved 7 left and 2 up.

New centre $(3-7,-5+2)=(-4,-3)$

$\therefore (x+4)^2+(y+3)^2=9$

Question 10

a
$$\begin{aligned} y &= \pm\sqrt{x} \\ &= \pm\sqrt{x} + 2 \\ y - 2 &= \pm\sqrt{x} \\ (y-2)^2 &= x \end{aligned}$$

b
$$\begin{aligned} y &= \pm\sqrt{x} \\ y &= \pm\sqrt{(x+4)} \\ y^2 &= x + 4 \end{aligned}$$

c
$$\begin{aligned} y &= \pm\sqrt{x} \\ y &= \pm\sqrt{(x-2)} + 1 \\ y - 1 &= \pm\sqrt{x-2} \\ (y-1)^2 &= x - 2 \end{aligned}$$

d
$$\begin{aligned} y &= \pm\sqrt{x} \\ y &= \pm\sqrt{(x-3)^2} - 2 \\ y + 2 &= \pm\sqrt{(x-3)^2} \\ (y+2)^2 &= (x-3)^2 \end{aligned}$$

Question 11

a
$$\begin{aligned} &A(3, 11) \quad B(12, -1) \\ &AB^2 = (12-3)^2 + (-1-11)^2 \\ &AB = 15 \end{aligned}$$

b Radius of circle centre A : 12

B : 3

Distance between centres = sum of radii.

$\therefore$ Circles are tangent to each other and have one point in common.

Question 12

a $C(2, 3) \quad D(-2, 5)$

$$CD^2 = (2-(-2))^2 + (3-5)^2$$

$$CD = \sqrt{20} = 2\sqrt{5} \approx 4.47$$

b Radius of circle centre C : 3

D : 1

Circle centres are further apart than sum of radii.

$\therefore$ No points in common.

Question 13

$(x-4)^2 + (y-2)^2 = 25$ & $y = x-3$

Solve simultaneously on classpad or graph and find points of intersection.

$$(x-4)^2 + (x-3-2)^2 = 25$$

$$x^2 - 8x + 16 + x^2 - 10x + 25 = 25$$

$$2x^2 - 18x + 16 = 0$$

$$2(x^2 - 9x + 8) = 0$$

$$2(x-8)(x-1) = 0$$

$\therefore x = 1 \quad$ or $\quad x = 8$

$y = -2 \qquad y = 5$

$\therefore$ coordinates $(1, -2)$ and $(8, 5)$

Question 14

$(x+5)^2 + (y-2)^2 = 34$ & $4y = x + 30$

$$x = 4y - 30$$

$$(4y - 30 + 5)^2 + (y-2)^2 = 34$$

$$16y^2 - 200y + 625 + y^2 - 4y + 4 = 34$$

$$17y^2 - 204y + 595 = 0$$

$$17(y^2 - 12y + 35) = 0$$

$$17(y-7)(y-5) = 0$$

$\therefore y = 5 \quad$ or $\quad y = 7$

$x = -10 \qquad x = -2$

$\therefore$ coordinates $(5, -10)$ and $(7, -2)$

Question 15

$(x-7)^2+(y-4)^2=40$ & $3y=x+25$

$$x=3y-25$$

$$(3y-25-7)^2+(y-4)^2=40$$

$$9y^2-192y+1024+y^2-8y+16=40$$

$$10y^2-200y+1040=40$$

$$10y^2-200y+1000=0$$

$$10(y^2-20y+100)=0$$

$$10(y-10)^2=0$$

$\therefore y=10,\ x=5$

As there is only one point of contact, (5, 10), the line $3y=x+25$ must be tangent to the circle.

Question 16

$$x^2+2x+y^2-10y+a=0$$

$$(x+1)^2-1+(y-5)^2-25+a=0$$

$$(x+1)^2+(y-5)^2-26+a=0$$

$$(x-1)^2+(y-5)^2=26-a$$

Radius $\sqrt{26-a}>0$

$\therefore a<26$

Question 1

a

$$y = a(x+3)(x-2)(x-4)$$
$$12 = a(0+3)(0-2)(0-4)$$
$$12 = 24a$$
$$a = \frac{1}{2}$$
$$\therefore \text{Equation } y = \frac{1}{2}(x+3)(x-2)(x-4)$$

b

$$y = a(x+2)^2(x-4)$$
$$-32 = a(0+2)^2(0-4)$$
$$-32 = -16a$$
$$a = 2$$
$$\therefore \text{Equation } y = 2(x+2)^2(x-4)$$

Question 2

$$y = x^2 - 4x - 6$$

a

$$x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times (-6)}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{40}}{2}$$
$$= \frac{4 \pm 2\sqrt{10}}{2}$$
$$= 2 \pm \sqrt{10}$$

b

$$x^2 - 4x - 6 = 0$$
$$(x-2)^2 - 4 - 6 = 0$$
$$(x-2)^2 - 10 = 0$$
$$(x-2)^2 = 10$$
$$x - 2 = \pm\sqrt{10}$$
$$x = 2 \pm \sqrt{10}$$

Question 3

$$x^2+6x+y^2-10y=15$$
$$(x+3)^2-9+(y-5)^2-25=15$$
$$(x-3)^2+(y-5)^2-25=15$$
$$(x-3)^2+(y-5)^2=49$$

Centre (3, 5) and radius 7

Question 4

a $f(4)=4$

b $g(4)=4^2=16$

c $h(4)=4^3=64$

d

$$p=p^2=p^3$$
$$\mathrm{p}\ =p^2$$
$$p^2-p\ =0$$
$$p\ (p-1)=0$$
$$p=0 \quad \text{or} \quad p=1$$

$$\mathrm{p}^3=p^2$$
$$p^3-p^2=0$$
$$p^2(p-1)=0$$
$$p=0 \quad \text{or} \quad p=1$$

All three functions have the same value when $p=0, p=1$.

Question 5

$\mathrm{f}_2(x)$ gradient 2.5, $\mathrm{f}_4(x)$ gradient -2

Question 6

$$5x + 2y = 9$$
$$2y = 9 - 5x$$
$$y = \frac{-5}{2}x + \frac{9}{2}$$
$$\therefore m = \frac{-5}{2}$$

Gradient of perpendicular line $m = \frac{2}{5}$.

$y = mx + c$ with $m = \frac{2}{5}$ passing through $(15, -1)$.

$$-1 = \frac{2}{5}(15) + c$$
$$-1 = 6 + c$$
$$c = -7$$
$$\therefore y = \frac{2}{5}x - 7$$

Question 7

a $x = -7$, $x = 2.25$, $x = 2.5$

b $x = -5.25$, $x = -1.5$, $x = 7$

c $x = 3$

d No real solutions.

Question 8

a $y = 5x$ Statements A, C

b $y = \frac{7}{x}$ Statements B, D

c $y = \frac{2}{x}$ Statements B, D

d $y = \frac{x}{3}$ Statements A, C

e $y = 2x + 1$ Statement A

f Statements A, C

g Statements B, D

h Statement B

Question 9

a $(2x-7)(x+9) = 0$

$$2x - 7 = 0 \quad \text{or} \quad x + 9 = 0$$
$$2x = 7$$
$$x = 3\frac{1}{2} \quad \text{or} \quad x = -9$$

b $x^2 - 8x + 12 = 0$

$$(x-6)(x-2) = 0$$
$$x - 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 6 \quad \text{or} \quad x = 2$$

c $5x^2 + 2x - 3 = 0$

$$(5x-3)(x+1) = 0$$
$$5x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$
$$5x = 3 \qquad x = -1$$
$$x = \frac{3}{5}$$

d $(x+11)(5x-4)(x-7)=0$

$x+11=0$ or $5x-4=0$ or $x-7=0$

$x=-11$ $\quad 5x=4$ $\quad x=7$

$x=\frac{4}{5}$

e $(x-3)(x^2+4x-5)=0$

$(x-3)(x+5)(x-1)=0$

$x-3=0$ or $x+5=0$ or $x-1=0$

$x=3$ $\quad x=-5$ $\quad x=1$

f $(x+5)(2x^2+x-6)=0$

$(x+5)(2x-3)(x+2)=0$

$x+5=0$ or $2x-3=0$ or $x+2=0$

$x=-5$ $\quad 2x=3$ $\quad x=-2$

$x=1.5$

Question 10

a Cubic

b Quadratic

c None of the listed types

d Cubic

e Reciprocal

f Linear

Question 11

$x^3 - 8x^2 + 19x - 12 = (x-3)(x^2 + bx + c)$

a $(-3)(+c) = -12$

$\therefore c = 4$

b $(x-3)(x^2 + bx + 4)$

$= x^3 + bx^2 + 4x - 3x^2 - 36x - 12$

$bx^2 - 3x^2 = 8x^2$

$b - 3 = -8$

$b = -5$

c $(x-3)(x^2 - 5x + 4)$

$= (x-3)(x-4)(x-1)$

Question 12

a $m = \dfrac{10}{x} \rightarrow mx = 10$

If x is doubled, m is halved to maintain balance.

b m and x are in inverse proportion.

If x is increased by a factor of k, m is decreased by a factor of k.

c $c = 20, \quad x = \dfrac{1}{2}$

d $\{x : x \in R,\ x \neq 0\}$

$\{y : y \in R,\ y \neq 0\}$

Question 13

In $\triangle ADB$, $\angle DAB = 49°, AB = 60,\ AD = 54$

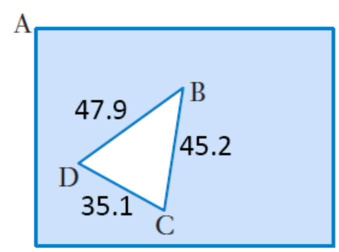
$$DB^2 = AB^2 + AD^2 - 2AB.AD\cos 49°$$
$$= 60^2 + 54^2 - 2 \times 60 \times 54 \times \cos 49°$$
$$DB = 47.6 \text{ mm}$$

In $\triangle ABC$, $\angle BAC = 32°, AB = 60,\ AC = 83$

$$BC^2 = AB^2 + AC^2 - 2AB.AC\cos 32°$$
$$= 60^2 + 83^2 - 2 \times 60 \times 83 \times \cos 32°$$
$$BC = 45.2 \text{ mm}$$

In $\triangle ADC$, $\angle DAC = 17°, AD = 54,\ AC = 83$

$$DC^2 = AD^2 + AC^2 - 2AD.AC\cos 17°$$
$$= 54^2 + 83^2 - 2 \times 54 \times 83 \times \cos 17°$$
$$DC = 35.1 \text{ mm}$$



Perimeter of $\triangle DBC$
$= 35.1 + 45.2 + 47.6$
$= 128$ mm

$$\cos\angle BDC = \frac{47.6^2 + 35.1^2 - 45.2^2}{2 \times 47.6 \times 35.1}$$
$$\angle BDC = 64.2°$$

$$\text{Area} = \frac{1}{2} \times 47.6 \times 35.1 \times \sin 64.2°$$
$$= 752 \text{ mm}^2$$

